10. ***REASONING*** The mass defect is the total mass of the stationary separated nucleons (protons and neutrons) minus the mass of the intact nucleus. The given atomic masses are for the electrical neutral atoms and, therefore, include the mass of the electrons. This will cause no problem, provided that we use the atomic mass of the hydrogen atom (including its electron) when determining the mass of the separated nucleons. Referring to Table 31.1 we find that the mass of a neutron is 1.008 665 u and the mass of a hydrogen atom is 1.007 825 u.

SOLUTION

a. The helium ${}^{3}_{2}\text{He}$ nucleus contains 2 protons and $3 - 2 = 1$ neutron. Thus, the mass defect Δm is

$$\Delta m = \underbrace{2(1.007\ 825\ \text{u}) + 1(1.008\ 665\ \text{u})}_{\text{Separated nucleons: 2 protons and 1 neutron}} - \underbrace{3.016\ 030\ \text{u}}_{\text{Intact nucleus}} = \boxed{0.008\ 285\ \text{u}}$$

b. The tritium ${}^{3}_{1}\text{T}$ nucleus contains 1 proton and $3 - 1 = 2$ neutrons. Thus, the mass defect Δm is

$$\Delta m = \underbrace{1(1.007\ 825\ \text{u}) + 2(1.008\ 665\ \text{u})}_{\text{Separated nucleons: 1 proton and 2 neutrons}} - \underbrace{3.016\ 050\ \text{u}}_{\text{Intact nucleus}} = \boxed{0.009\ 105\ \text{u}}$$

c. The mass defect for tritium ${}^{3}_{1}\text{T}$ is greater than that for helium ${}^{3}_{2}\text{He}$. The mass defect is related to the binding energy as follows:

$$\text{Binding energy} = (\Delta m)c^2 \tag{31.3}$$

The binding energy is the energy that must be supplied to the intact nucleus in order to separate it into its constituent nucleons. Thus,

$$\boxed{\text{more energy must be supplied to tritium } {}^{3}_{1}\text{T} \text{ than to helium } {}^{3}_{2}\text{He}}.$$

12. ***REASONING*** To obtain the binding energy, we will calculate the mass defect and then use the fact that 1 u is equivalent to 931.5 MeV. The atomic mass given for ${}^{7}_{3}\mathrm{Li}$ includes the 3 electrons in the neutral atom. Therefore, when computing the mass defect, we must account for these electrons. We do so by using the atomic mass of 1.007 825 u for the hydrogen atom ${}^{1}_{1}\mathrm{H}$, which also includes the single electron, instead of the atomic mass of a proton.

SOLUTION Noting that the number of neutrons is 7 – 3 = 4, we obtain the mass defect Δm as follows:

$$\Delta m = \underbrace{3(1.007\,825\text{ u})}_{\substack{\text{3 hydrogen atoms}\\ \text{(protons plus electrons)}}} + \underbrace{4(1.008\,665\text{ u})}_{\text{4 neutrons}} - \underbrace{7.016\,003\text{ u}}_{\substack{\text{Intact lithium atom}\\ \text{(including 3 electrons)}}} = 4.2132\times10^{-2}\text{ u}$$

Since 1 u is equivalent to 931.5 MeV, the binding energy is

$$\text{Binding energy} = \left(4.2132\times10^{-2}\text{ u}\right)\left(\frac{931.5\text{ MeV}}{1\text{ u}}\right) = \boxed{39.25\text{ MeV}}$$

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14. ***REASONING*** The mass defect Δm is related to the total binding energy and the square of the speed c of light. Thus, to determine Δm we will use Figure 31.5 to obtain the binding energy per nucleon for the oxygen ${}^{16}_{8}O$ nucleus and multiply this value by the total number of nucleons (16) to find the total binding energy. Note that the data in Figure 31.5 is given in MeV per nucleon and that an MeV is not an SI unit compatible with kilograms. Thus, we will need to convert MeV to joules (J).

SOLUTION The mass defect Δm is given by

$$\Delta m = \frac{\text{Binding energy}}{c^2} \qquad (31.3)$$

In Figure 31.5 we can see that the binding energy per nucleon for oxygen ${}^{16}_{8}O$ is 8.00 MeV per nucleon. Therefore, the total binding energy for the 16 nucleons in the nucleus is

$$\text{Binding energy} = (8.00\text{ MeV/nucleon})(16\text{ nucleons}) = 128\text{ MeV}$$

Using this value in Equation 31.3 and converting the energy units of MeV into joules (J), we find that

$$\Delta m = \frac{\text{Binding energy}}{c^2} = \frac{(128\text{ MeV})\left(\dfrac{1\times10^{6}\text{ eV}}{1\text{ MeV}}\right)\left(\dfrac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)}{\left(3.00\times10^{8}\text{ m/s}\right)^2} = \boxed{2.28\times10^{-28}\text{ kg}}$$

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17. **SSM** ***REASONING AND SOLUTION*** The general form for β^- decay is

$$\underbrace{{}^{A}_{Z}\text{P}}_{\substack{\text{Parent}\\ \text{nucleus}}} \rightarrow \underbrace{{}^{A}_{Z+1}\text{D}}_{\substack{\text{Daughter}\\ \text{nucleus}}} + \underbrace{{}^{0}_{-1}\text{e}}_{\substack{\beta^-\text{ particle}\\ \text{(electron)}}}$$

Therefore, the β^- decay process for ${}^{35}_{16}\text{S}$ is $\boxed{{}^{35}_{16}\text{S} \rightarrow {}^{35}_{17}\text{Cl} + {}^{0}_{-1}\text{e}}$.

18. ***REASONING AND SOLUTION***

a. The decay reaction is: ${}^{242}_{94}\text{Pu} \rightarrow {}^{A}_{Z}\text{X} + {}^{4}_{2}\text{He}$. Therefore, $242 = A + 4$, so that $A = 238$. In addition, $94 = Z + 2$, so that $Z = 92$. Thus, the daughter nucleus is $\boxed{{}^{238}_{92}\text{U}}$.

b. The decay reaction is ${}^{24}_{11}\text{Na} \rightarrow {}^{A}_{Z}\text{X} + {}^{0}_{-1}\text{e}$. Therefore, $24 = A$. In addition, $11 = Z - 1$, so that $Z = 12$. Thus the daughter nucleus is $\boxed{{}^{24}_{12}\text{Mg}}$.

c. The decay reaction is ${}^{13}_{7}\text{N} \rightarrow {}^{A}_{Z}\text{X} + {}^{0}_{1}\text{e}$. Therefore, $13 = A$. In addition, $7 = Z + 1$, so that $Z = 6$. Thus, the daughter nucleus is $\boxed{{}^{13}_{6}\text{C}}$

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29. ***REASONING*** The half-life $T_{1/2}$ is inversely related to the decay constant λ by the relation $T_{1/2} = 0.693/\lambda$ (Equation 31.6). To convert from seconds to days, we use the fact that 3600 s = 1 h and 24 h = 1 day.

SOLUTION The half-life of radium ${}^{224}_{88}\text{Ra}$ is

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{\left(2.19\times10^{-6}\ \text{s}^{-1}\right)\left(\dfrac{3600\ \text{s}}{1\ \text{h}}\right)\left(\dfrac{24\ \text{h}}{1\ \text{day}}\right)} = \boxed{3.66\ \text{days}}$$

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32. ***REASONING AND SOLUTION*** The amount remaining is 0.0100% = 0.000 100. We know $N/N_0 = e^{-0.693t/T_{1/2}}$. Therefore, we find

$$t = -\frac{T_{1/2}}{0.693}\ln\left(\frac{N}{N_0}\right) = -\frac{29.1\text{ yr}}{0.693}\ln(0.000\,100) = \boxed{387\text{ yr}}$$

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33. ***REASONING AND SOLUTION*** The activity is $A = \lambda N$. The decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(5.27\text{ yr})\left(\dfrac{3.156\times10^{7}\text{ s}}{1\text{ yr}}\right)} = 4.17\times10^{-9}\text{ s}^{-1}$$

As discussed in Section 14.1, the number N of nuclei is the number of moles of nuclei times Avogadro's number (which is the number of nuclei per mole). Thus,

$$N = \underbrace{\left(\frac{0.50\text{ g}}{59.9\text{ g/mol}}\right)}_{\text{Number of moles}}\underbrace{\left(6.02\times10^{23}\text{ mol}^{-1}\right)}_{\text{Avogadro's number}} = 5.0\times10^{21}$$

Therefore, $A = \lambda N = (4.17\times10^{-9}\text{ s}^{-1})(5.0\times10^{21}) = \boxed{2.1\times10^{13}\text{ Bq}}$.

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34. ***REASONING AND SOLUTION*** According to Equation 31.5, the fraction of an initial sample remaining after a time t is $N/N_0 = e^{-\lambda t}$, where λ is the decay constant. The decay constant is related to the half-life $T_{1/2}$. According to Equation 31.6, the decay constant is $\lambda = 0.693/T_{1/2}$. Therefore, the fraction remaining is

$$\frac{N}{N_0} = e^{-0.693t/T_{1/2}} = e^{-0.693\left[(30.0\text{ days})/(8.04\text{ days})\right]} = 0.0753$$

This fraction corresponds to a percentage of $\boxed{7.53\ \%}$.

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40. ***REASONING*** In the radiocarbon method, the number of radioactive nuclei remaining at a given instant is related to the number present initially, the time that has passed since the individual died, and the decay constant for ${}^{14}_{6}\text{C}$. Thus, to determine how long ago the individual died, we will need information about the number of nuclei present in the material discovered with the mummy and the number present initially, which can be related to the activity of the material found with the body and the initial activity. We will also need the decay constant, which can be obtained from the half-life of ${}^{14}_{6}\text{C}$.

SOLUTION The number N of radioactive nuclei present at a time t is

$$N = N_0 e^{-\lambda t} \qquad (31.5)$$

where N_0 is the number present initially at $t = 0$ s and λ is the decay constant for ${}^{14}_{6}\text{C}$. Solving this equation for t, we find that

$$\frac{N}{N_0} = e^{-\lambda t} \qquad \text{or} \qquad \ln\left(\frac{N}{N_0}\right) = -\lambda t \qquad \text{or} \qquad t = -\left(\frac{1}{\lambda}\right)\ln\left(\frac{N}{N_0}\right)$$

Since the activity A is proportional to the number N of radioactive nuclei, this expression for t becomes

$$t = -\left(\frac{1}{\lambda}\right)\ln\left(\frac{N}{N_0}\right) = -\left(\frac{1}{\lambda}\right)\ln\left(\frac{A}{A_0}\right) \qquad (1)$$

The decay constant is related to the half-life $T_{1/2}$ according to

$$\lambda = \frac{0.693}{T_{1/2}} \qquad (31.6)$$

Substituting this expression into Equation (1) reveals that

$$t = -\left(\frac{1}{\lambda}\right)\ln\left(\frac{A}{A_0}\right) = -\left(\frac{1}{0.693/T_{1/2}}\right)\ln\left(\frac{A}{A_0}\right)$$

Noting that $A/A_0 = 0.785$ and that $T_{1/2} = 5730$ yr, we find that

$$t = -\left(\frac{T_{1/2}}{0.693}\right)\ln\left(\frac{A}{A_0}\right) = -\left(\frac{5730\text{ yr}}{0.693}\right)\ln(0.785) = \boxed{2.00\times 10^3\text{ yr}}$$