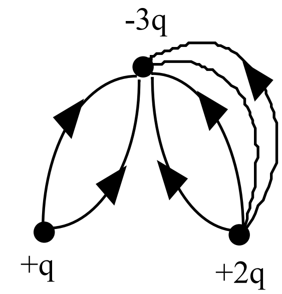
25. ***REASONING AND SOLUTION*** The electric field lines must originate on the positive charges and terminate on the negative charge. They cannot cross one another. Furthermore, the number of field lines beginning or terminating on any charge must be proportional to the magnitude of the charge. Thus, for every field line that leaves the charge +q, two field lines must leave the charge +2q. These three lines must terminate on the −3q charge. If the sketch is to have six field lines, two of them must originate on +q, and four of them must originate on the charge +2q.



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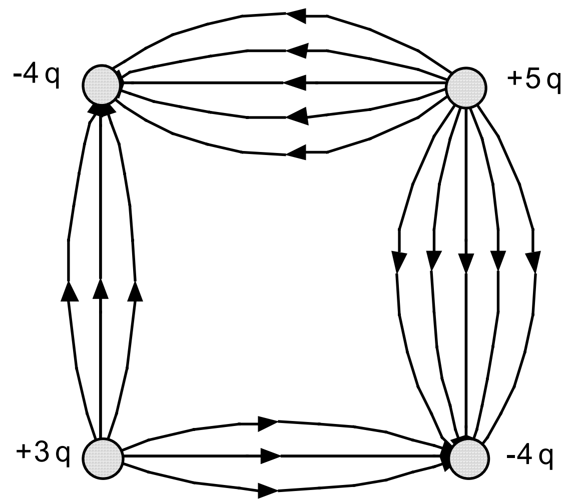
26. ***REASONING AND SOLUTION***
The electric field lines must originate on the positive charges and terminate on the negative charges. They cannot cross one another. Furthermore, the number of field lines beginning or ending on any charge must be proportional to the magnitude of the charge. If 10 electric field lines leave the +5q charge, then six lines must originate from the +3q charge, and eight lines must end on each –4q charge. The drawing shows the electric field lines that meet these criteria.



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27. SSM WWW ***REASONING*** Two forces act on the charged ball (charge q); they are the downward force of gravity $m\mathbf{g}$ and the electric force **F** due to the presence of the charge q in the electric field **E**. In order for the ball to float, these two forces must be equal in magnitude and opposite in direction, so that the net force on the ball is zero (Newton's second law). Therefore, **F** must point upward, which we will take as the positive direction. According to Equation 18.2, $\mathbf{F} = q\mathbf{E}$. Since the charge q is negative, the electric field **E** must point downward, as the product $q\mathbf{E}$ in the expression $\mathbf{F} = q\mathbf{E}$ must be positive, since the force **F** points upward. The magnitudes of the two forces must be equal, so that $mg = |q|E$. This expression can be solved for E.

SOLUTION The magnitude of the electric field **E** is

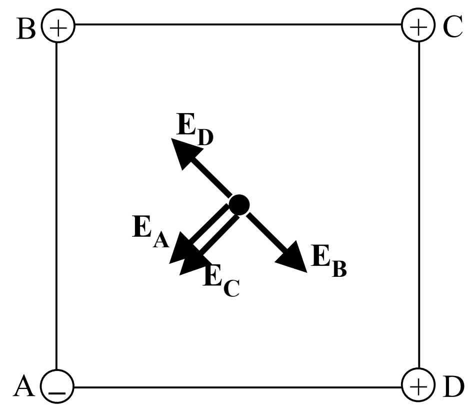
$$E = \frac{mg}{|q|} = \frac{(0.012\text{ kg})(9.80\text{ m/s}^2)}{18\times10^{-6}\text{ C}} = \boxed{6.5\times10^{3}\text{ N/C}}$$

As discussed in the reasoning, this electric field points $\boxed{\text{downward}}$.

---

28. ***REASONING*** Each charge creates an electric field at the center of the square, and the four fields must be added as vectors to obtain the net field. Since the charges all have the same magnitude and since each corner is equidistant from the center of the square, the magnitudes of the four individual fields are identical. Each is given by Equation 18.3 as $E = \frac{k|q|}{r^2}$. The directions of the various contributions are not the same, however. The field created by a positive charge points away from the charge, while the field created by a negative charge points toward the charge.

SOLUTION The drawing at the right shows each of the field contributions at the center of the square (see black dot). Each is directed along a diagonal of the square. Note that $\mathbf{E_D}$ and $\mathbf{E_B}$ point in opposite directions and, therefore, cancel, since they have the same magnitude. In contrast $\mathbf{E_A}$ and $\mathbf{E_C}$ point in the same direction toward corner A and, therefore, combine to give a net field that is twice the magnitude of $\mathbf{E_A}$ or $\mathbf{E_C}$. In other words, the net field at the center of the square is given by the following vector equation:



$$\Sigma\mathbf{E} = \mathbf{E_A} + \mathbf{E_B} + \mathbf{E_C} + \mathbf{E_D} = \mathbf{E_A} + \mathbf{E_B} + \mathbf{E_C} - \mathbf{E_B} = \mathbf{E_A} + \mathbf{E_C} = 2\mathbf{E_A}$$

Using Equation 18.3, we find that the magnitude of the net field is

$$\Sigma E = 2E_A = 2\frac{k|q|}{r^2}$$

In this result r is the distance from a corner to the center of the square, which is one half of the diagonal distance d. Using L for the length of a side of the square and taking advantage of the Pythagorean theorem, we have $r = \frac{1}{2}d = \frac{1}{2}\sqrt{L^2 + L^2}$. With this substitution for r, the magnitude of the net field becomes

$$\Sigma E = 2\frac{k|q|}{\left(\frac{1}{2}\sqrt{L^2 + L^2}\right)^2} = \frac{4k|q|}{L^2} = \frac{4\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.4\times10^{-12}\ \text{C}\right)}{(0.040\ \text{m})^2} = \boxed{54\ \text{N/C}}$$

29. ***REASONING AND SOLUTION***

a. In order for the field to be zero, the point cannot be between the two charges. Instead, it must be located on the line between the two charges on the side of the positive charge and away from the negative charge. If x is the distance from the positive charge to the point in question, then the negative charge is at a distance (3.0 m + x) meters from this point. For the field to be zero here we have

$$\frac{k|q_-|}{(3.0\text{ m}+x)^2}=\frac{k|q_+|}{x^2} \quad \text{or} \quad \frac{|q_-|}{(3.0\text{ m}+x)^2}=\frac{|q_+|}{x^2}$$

Solving for the ratio of the charge magnitudes gives

$$\frac{|q_-|}{|q_+|}=\frac{16.0\ \mu\text{C}}{4.0\ \mu\text{C}}=\frac{(3.0\text{ m}+x)^2}{x^2} \quad \text{or} \quad 4.0=\frac{(3.0\text{ m}+x)^2}{x^2}$$

Suppressing the units for convenience and rearranging this result gives

$$4.0x^2=(3.0+x)^2 \quad \text{or} \quad 4.0x^2=9.0+6.0x+x^2 \quad \text{or} \quad 3x^2-6.0x-9.0=0$$

Solving this quadratic equation for x with the aid of the quadratic formula (see Appendix C.4) shows that

$$x=3.0\text{ m} \quad \text{or} \quad x=-1.0\text{ m}$$

We choose the positive value for x, since the negative value would locate the zero-field spot between the two charges, where it can not be (see above). Thus, we have $\boxed{x = 3.0\text{ m}}$.

b. Since the field is zero at this point, the force acting on a charge at that point is $\boxed{0\text{ N}}$.

---

30. ***REASONING***

a. The magnitude E of the electric field is given by $E = \sigma / \varepsilon_0$ (Equation 18.4), where σ is the charge density (or charge per unit area) and ε_0 is the permittivity of free space.

b. The magnitude F of the electric force that would be exerted on a Na^+ ion placed inside the membrane is the product of the magnitude $|q_0|$ of the charge and the magnitude E of the electric field (see Equation 18.2), or $F = |q_0| E$.

SOLUTION

a. The magnitude of the electric field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{7.1\times10^{-6}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)} = \boxed{8.0\times10^{5}\ \text{N/C}}$$

b. The magnitude F of the force exerted on a Na^+ ion ($q_0 = +e$) is

$$F = |q_0| E = |e| E = \left|1.60\times10^{-19}\ \text{C}\right|\left(8.0\times10^{5}\ \text{N/C}\right) = \boxed{1.3\times10^{-13}\ \text{N}}$$

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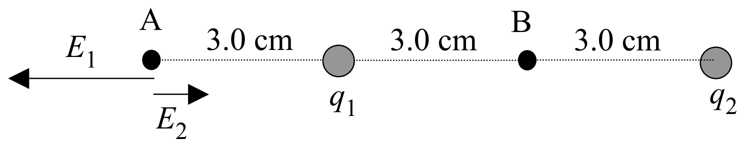
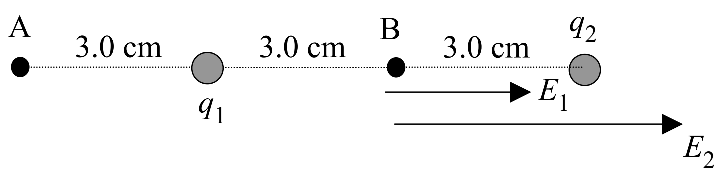
31. SSM ***REASONING***

a. The drawing shows the two point charges q_1 and q_2. Point A is located at $x = 0$ cm, and point B is at $x = +6.0$ cm.



Since q_1 is positive, the electric field points away from it. At point A, the electric field E_1 points to the left, in the $-x$ direction. Since q_2 is negative, the electric field points toward it. At point A, the electric field E_2 points to the right, in the $+x$ direction. The net electric field is $E = -E_1 + E_2$. We can use Equation 18.3, $E = k|q|/r^2$, to find the magnitude of the electric field due to each point charge.

b. The drawing shows the electric field produced by the charges q_1 and q_2 at point B, which is located at $x = +6.0$ cm.



Since q_1 is positive, the electric field points away from it. At point B, the electric field points to the right, in the $+x$ direction. Since q_2 is negative, the electric field points toward it. At point B, the electric field points to the right, in the $+x$ direction. The net electric field is $E = +E_1 + E_2$.

SOLUTION

a. The net electric field at the origin (point A) is $E = -E_1 + E_2$:

$$E = -E_1 + E_2 = \frac{-k|q_1|}{r_1^2} + \frac{k|q_2|}{r_2^2}$$

$$= \frac{-\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.5\times10^{-6}\ \text{C}\right)}{\left(3.0\times10^{-2}\ \text{m}\right)^2} + \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(21\times10^{-6}\ \text{C}\right)}{\left(9.0\times10^{-2}\ \text{m}\right)^2}$$

$$= \boxed{-6.2\times10^7\ \text{N/C}}$$

The minus sign tells us that the net electric field points along the $-x$ axis.

b. The net electric field at $x = +6.0$ cm (point B) is $E = E_1 + E_2$:

$$E = E_1 + E_2 = \frac{k|q_1|}{r_1^2} + \frac{k|q_2|}{r_2^2}$$

$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.5\times10^{-6}\ \text{C})}{(3.0\times10^{-2}\ \text{m})^2} + \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(21\times10^{-6}\ \text{C})}{(3.0\times10^{-2}\ \text{m})^2}$$

$$= \boxed{+2.9\times10^8\ \text{N/C}}$$

The plus sign tells us that the net electric field points along the $+x$ axis.

34. ***REASONING AND SOLUTION*** The figure at the right shows the configuration given in text Figure 18.21*a*. The electric field at the center of the rectangle is the resultant of the electric fields at the center due to each of the four charges. As discussed in Conceptual Example 11, the magnitudes of the electric field at the center due to each of the four charges are equal. However, the fields produced by the charges in corners 1 and 3 are in opposite directions. Since they have the same magnitudes, they combine to give zero resultant.

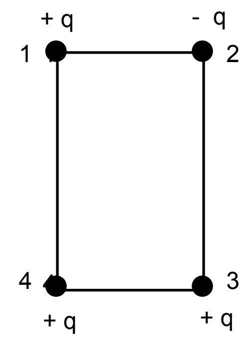


Figure 1

The fields produced by the charges in corners 2 and 4 point in the same direction (toward corner 2). Thus, $E_C = E_{C2} + E_{C4}$, where E_C is the magnitude of the electric field at the center of the rectangle, and E_{C2} and E_{C4} are the magnitudes of the electric field at the center due to the charges in corners 2 and 4 respectively. Since both E_{C2} and E_{C4} have the same magnitude, we have $E_C = 2\,E_{C2}$.

The distance *r*, from any of the charges to the center of the rectangle, can be found using the Pythagorean theorem:

$$d = \sqrt{(3.00\text{ cm})^2 + (5.00\text{ cm})^2} = 5.83\text{ cm}$$

Therefore, $r = \dfrac{d}{2} = 2.92\text{ cm} = 2.92\times10^{-2}\text{ m}$

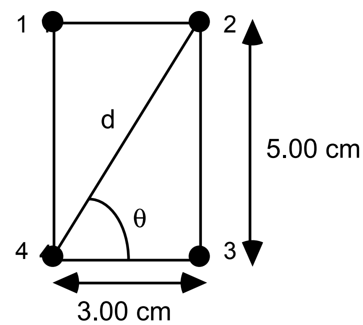


Figure 2

The electric field at the center has a magnitude of

$$E_C = 2E_{C2} = \frac{2k|q_2|}{r^2} = \frac{2(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(8.60\times10^{-12}\text{ C})}{(2.92\times10^{-2}\text{ m})^2} = \boxed{1.81\times10^2\text{ N/C}}$$

The figure at the right shows the configuration given in text Figure 18.21*b.* All four charges contribute a non-zero component to the electric field at the center of the rectangle. As discussed in Conceptual Example 11, the contribution from the charges in corners 2 and 4 point toward corner 2 and the contribution from the charges in corners 1 and 3 point toward corner 1.

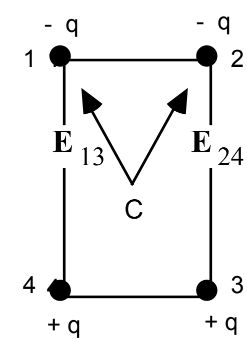


Figure 3

Notice also, the magnitudes of E_{24} and E_{13} are equal, and, from the first part of this problem, we know that

$$E_{24} = E_{13} = 1.81 \times 10^2 \text{ N/C}$$

The electric field at the center of the rectangle is the vector sum of $\mathbf{E_{24}}$ and $\mathbf{E_{13}}$. The *x* components of $\mathbf{E_{24}}$ and $\mathbf{E_{13}}$ are equal in magnitude and opposite in direction; hence

$$(E_{13})_x - (E_{24})_x = 0$$

Therefore,

$$E_\text{C} = (E_{13})_y + (E_{24})_y = 2(E_{13})_y = 2(E_{13})\sin\theta$$

From Figure 2, we have that

$$\sin\theta = \frac{5.00 \text{ cm}}{d} = \frac{5.00 \text{ cm}}{5.83 \text{ cm}} = 0.858$$

and

$$E_\text{C} = 2\left(E_{13}\right)\sin\theta = 2\left(1.81\times10^2 \text{ N/C}\right)\left(0.858\right) = \boxed{3.11\times10^2 \text{ N/C}}$$

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35. ***REASONING AND SOLUTION*** The magnitude of the force on q_1 due to q_2 is given by Coulomb's law:

$$F_{12} = \frac{k|q_1||q_2|}{r_{12}^{\ 2}} \qquad (1)$$

The magnitude of the force on q_1 due to the electric field of the capacitor is given by

$$F_{1\text{C}} = |q_1|E_\text{C} = |q_1|\left(\frac{\sigma}{\varepsilon_0}\right) \qquad (2)$$

Equating the right hand sides of Equations (1) and (2) above gives

$$\frac{k|q_1||q_2|}{r_{12}^{\ 2}} = |q_1|\left(\frac{\sigma}{\varepsilon_0}\right)$$

Solving for r_{12} gives

$$r_{12} = \sqrt{\frac{\varepsilon_0 k|q_2|}{\sigma}}$$

$$= \sqrt{\frac{[8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)](8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\ \text{C})}{(1.30\times10^{-4}\ \text{C/m}^2)}} = \boxed{5.53\times10^{-2}\ \text{m}}$$

---

37. ***REASONING*** The electric field is given by Equation 18.2 as the force **F** that acts on a test charge q_0, divided by q_0. Although the force is not known, the acceleration and mass of the charged object are given. Therefore, we can use Newton's second law to determine the force as the mass times the acceleration and then determine the magnitude of the field directly from Equation 18.2. The force has the same direction as the acceleration. The direction of the field, however, is in the direction opposite to that of the acceleration and force. This is because the object carries a negative charge, while the field has the same direction as the force acting on a positive test charge.

SOLUTION According to Equation 18.2, the magnitude of the electric field is

$$E = \frac{F}{|q_0|}$$

According to Newton's second law, the net force acting on an object of mass m and acceleration a is $\Sigma F = ma$. Here, the net force is the electrostatic force F, since that force alone acts on the object. Thus, the magnitude of the electric field is

$$E = \frac{F}{|q_0|} = \frac{ma}{|q_0|} = \frac{\left(3.0\times10^{-3}\text{ kg}\right)\left(2.5\times10^{3}\text{ m/s}^2\right)}{34\times10^{-6}\text{ C}} = \boxed{2.2\times10^{5}\text{ N/C}}$$

The direction of this field is opposite to the direction of the acceleration. Thus, the field points $\boxed{\text{along the }-x\text{ axis}}$.
