

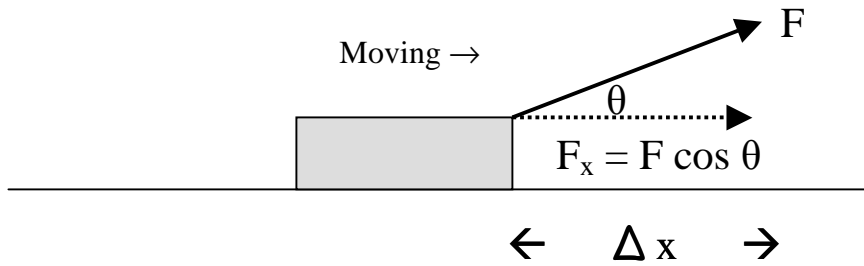
Ch6: Work and Energy

Recall from Ch4: Force: any agent of *change*.

Energy, \tilde{E} , is the capacity a body or system has to *cause* change.

Work is Energy transferred to (+) or removed from (-) a system or body by means of a force acting on it.

Work and Energy are scalars.



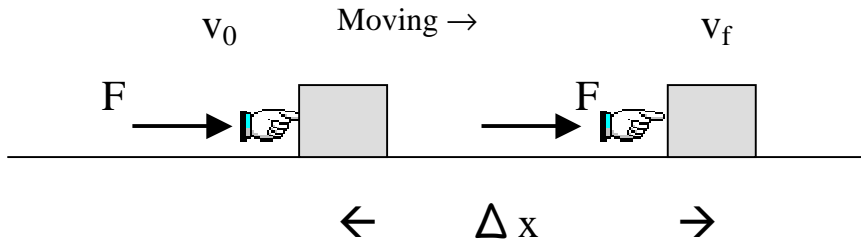
The Work done by the force F on the box is:

Work = (the component of the force along the direction of motion) * (the distance thru which that component acts)

$$W = F_x \Delta x \quad (\text{units??})$$

$$W = F \cos \theta \Delta x = F \Delta x \cos(\vec{F}, \Delta \vec{x})$$

For a constant force F :



$$W = F_x \Delta x \quad \text{but } F = ma$$

So $W = ma \Delta x$ use the 3rd eq of motion...

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

The quantity $\frac{1}{2}mv^2$ is called the **KINETIC ENERGY**.

$$K = \frac{1}{2}mv^2$$

KE is the energy a body or system has due to its motion.

UNITS??

Note: $KE \geq 0$ why?

$$\text{So, } W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2$$

becomes $W = K_f - K_o$

or $W_{\text{net}} = \Delta K$

Work-Energy Theorem

Work-Energy Theorem :

The net work
done by all
external
forces

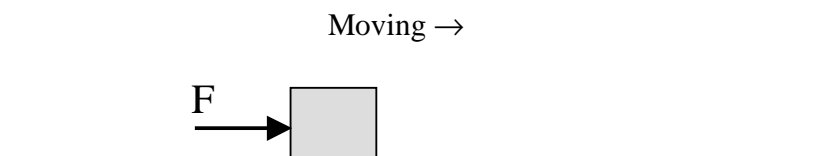
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The change in
Kinetic Energy

Or, more intuitively:

$$K_o = K_f + W_{\text{net}}$$

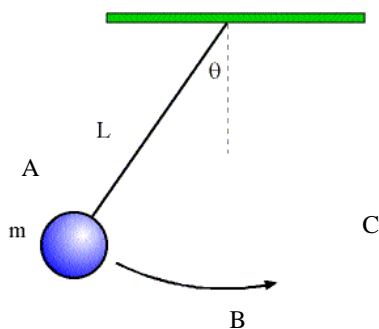
Thus, when a force **F** does work on an object, that object will acquire a KE of $\frac{1}{2} m v^2$.



Recap:

Energy: the capacity a body has to cause change.

Work is Energy transferred to or from a body or system by means of a force acting on it.



1. You did work to lift the ball to 'A' by applying a force F .
2. The ball has acquired some form of Energy. (Energy has been transferred to the ball.)
3. Now let go of the ball.
4. The ball speeds up from $A \rightarrow B$ because gravity is doing *positive work*.
5. The ball slows down from $B \rightarrow C$ because gravity is doing *negative work*.

So we'll see that Work can be both + or -.

Consider a system of objects:

{ Ball + ____?____ }



Let's slowly lift the ball from point A to point B.

$$W_{\text{net}} = \Delta K$$

$$W_{\text{net}} = W_{\text{gravity}} + W_{\text{me}} = -mgh + mgh = 0$$

$$\text{So } W_{\text{net}} = \Delta K = 0.$$

But WORK is the transfer of Energy.

$W_{me} = mgh$, but in this case the Work done by me did not change the ball's Kinetic Energy, rather the W_{me} is now stored as Gravitational Potential Energy in the Ball-Earth system

GPE: the energy a body has due to its position in a gravitational field.

The ball's Potential Energy at B is called a POTENTIAL energy because that energy is 'potentially' available to be converted/transformed into other forms of energy (...when/how?...)

Potential Energy is an energy due to configuration:

Potential Energy only has meaning w/ respect to some reference point.

Thus only *changes* in Potential Energy have significance.

Let's pick $PE = 0$ at point A.

So now $\Delta PE_{AB} = mgh$, and we can say:

$PE_{\text{ball}} = mgh$ at point B.

(*It's really the PE of the ball-Earth system. But since only the ball moves we can say "the PE of the ball". Without the Earth, the ball *has no* PE.)

Now drop the ball:

$$W_{\text{gravity},BA} = mgh, \quad \text{but} \quad \Delta PE = -mgh$$

So $W_{\text{gravity}} = -\Delta PE$ (The GPE decreases)

This is true for any Conservative Force.

Gravity is a Conservative Force.

A force is said to be *conservative* if the work done by that force in moving a body from point $A \rightarrow B$ is independent of the path taken from $A \rightarrow B$.

Since W_{gravity} is independent of the path from $A \rightarrow B$, there is some property of the ball-Earth system that depends only on the initial and final positions of the ball.

This property is the GPE.

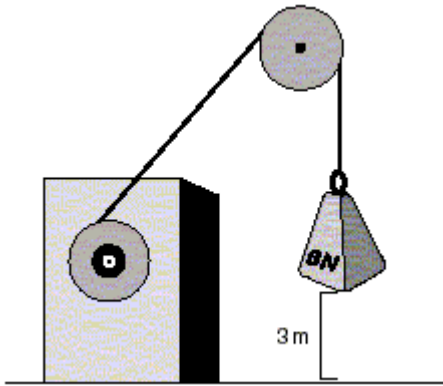
A force is said to be conservative if the work done by that force can be stored as a potential energy and be fully recoverable (you can get all of the work 'back' as mechanical energy).

Thus, only conservative forces have potential energy functions!

What are some conservative and non-conservative forces?

Also: $\oint W_{\text{conservative}} = 0$.

POWER



Not only do we want to know how much work is needed to lift an object, but we also may want to know *how fast* work is

being done (how fast is energy being delivered/transferred/used...).

Power: the rate at which Work is done, or the rate at which energy is transferred.

$$P_{\text{average}} = \frac{\text{Energy}}{\text{time}} = \frac{\text{Work done}}{\text{time}}$$

$$\text{Units?} \quad \frac{\text{Energy}}{\text{time}} \rightarrow \frac{\text{Joules}}{\text{second}} = \text{Watt, W}$$

$$\text{So } 1 \text{ Watt} = 1 \text{ J/s}$$

$$\overline{Power} = \frac{W}{t}, \text{ and } W = F_x \Delta x$$

$$\text{then } \overline{P} = \frac{F_x \Delta x}{\Delta t} = F_x \overline{v}$$



A motor lifts a 6000-N piano (~ 1300 lbs!) to a height of 8 floors (~ 30 m) in 4 minutes. What is the minimum power that the motor must deliver? (1 hp ~ 746 W)

What do you purchase from  ?

~~More on~~ More on Energy

When a conservative force does work on an object w/in a system, that force transfers Energy between the Kinetic Energy of the object and the Potential Energy of the system.

Ex:

Consider a system of objects:

{ Ball + ____?____ }

Drop the ball from point A and let it fall to point B.

* As gravity does positive work:

- the PE decreases while
- the KE increases, $W = \Delta K$

$W = \Delta K$ the work done by our conservative force
(gravity).

And recall for a conservative force;


$W = -\Delta PE$, so equate $W = W$ to get

$$\Delta K = -\Delta PE$$

or $\Delta K + \Delta PE = 0$

$$(K_f - K_o) + (PE_f - PE_o) = 0$$

$$(K_f + PE_f) - (K_o + PE_o) = 0$$


$$E_f - E_o = 0, \text{ where}$$

"E" is the Total Mechanic Energy; $E = K + PE$

so $\Delta E = 0$ or $E_o = E_f$

IF $E_o = E_f$, that means that the total mechanical Energy does not change (it is conserved).

Law of Conservation of Energy!

If only conservative forces act within a system, then the total mechanical energy cannot change, it is conserved.

"Energy cannot be created or destroyed, it can only be transformed."

This is the 'Law of Conservation of Mechanical Energy'.

As the ball falls to Earth, the work done by gravity is converted into kinetic energy (at the expense of reducing the ball's potential energy).

So the total mechanic energy E of the ball (really the ball-Earth system) remains unchanged.

$KE_{\text{increases}}$ while the $PE_{\text{decreases}}$

Law of Conservation of Mechanical Energy

- * it holds only when conservative forces act (no friction, fingers, strings, wind,...)
- * conservative forces (gravity, springs...) lead to conservation principles.

Here's another way of looking at the falling ball problem:

Analysis:

Ch2	Ch6	Ch6
Kinematic Eq's of motion	Work-Energy Th^m	Conservation of Energy

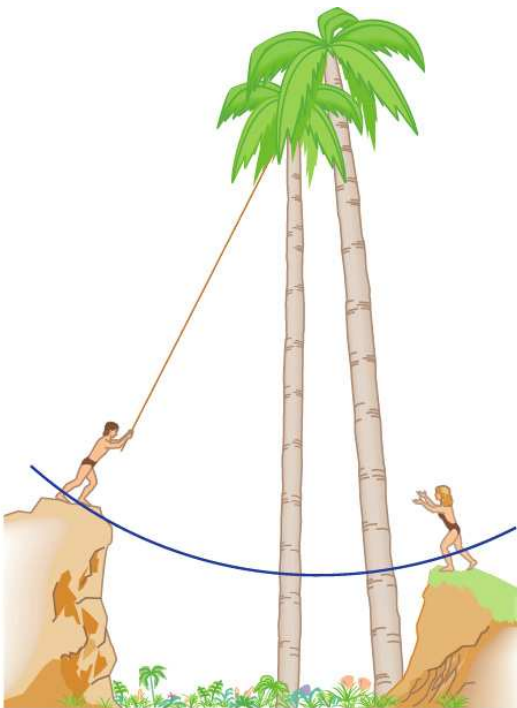
When applying $E_o = E_f$, you will need to pick a reference point for PE.

Rule of  :

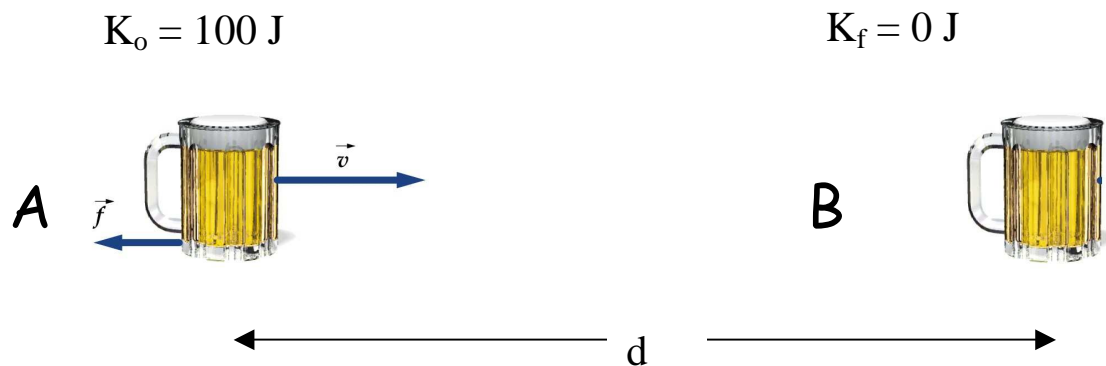
Let PE = zero at some convenient place.

A good choice is to pick PE = 0 at the lowest position that the object will/can be.

Tarzan, who weighs 800.0 Newtons, is standing on the edge of a cliff while holding on to the end of a convenient 15.00-meter vine. From the top of the cliff to the bottom of the swing, Tarzan would fall by 2.60 meters. The vine has a breaking strength of 1112 Newtons. Will the vine break?



Accounting for Nonconservative Forces



100 Joules of mechanical energy were dissipated by f_k . Where did it go?

Do you think the Work done by friction is independent of the path?

The thermal energy (or internal energy) produced is not a mechanical potential energy.

It's not a reversible process.
Thus, f_k is a nonconservative force.

So we can say that the Work done by the nonconservative force f_k is -100J .

$$W_{nc} = -100\text{J}.$$

f_k has no associated potential energy function (since the work done by f_k is not stored as a potential energy).

We have $\Delta K = -100 \text{ J}$, and since $\Delta PE = 0$

$$\Delta K + \Delta PE = -100\text{J} \quad \text{*also: } W_{nc} = -100\text{J}$$

Is $\Delta E = 0$? (is Energy conserved??)

Actually $\Delta E = -100\text{J}$,

Now we can modify our $E_f = E_o$ expression to include:

- a) losses due to the nonconservative force f_k
- b) all other nonconservative external forces that may be doing work (ex: fingers, ropes...)

$$E_f = E_o \quad \text{or} \quad \Delta E = 0$$

But when nonconservative forces act they can decrease E or increase E (examples?)

So we can now modify our $\Delta E = 0$ to be:

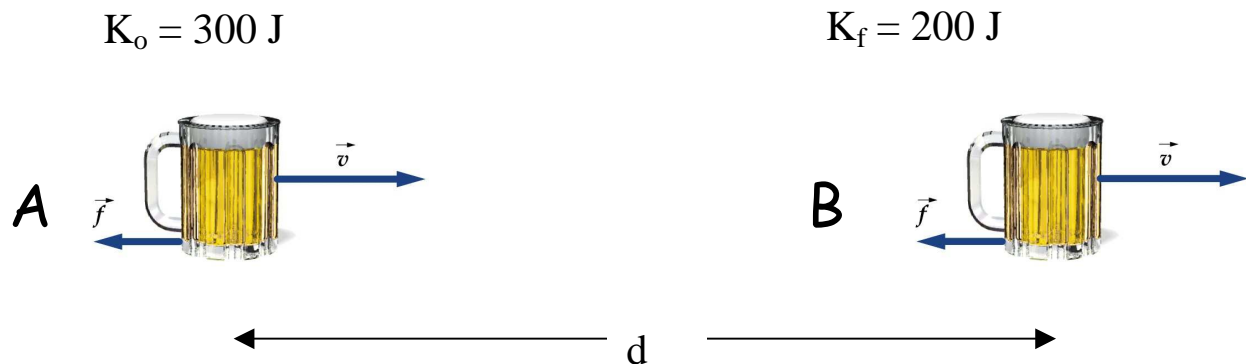
$$\Delta E = W_{nc}$$

Where W_{nc} is the work done by all nonconservative forces.

Accounting 101:

We will be using the concept of ENERGY as a bookkeeping tool.

Let's say:



Is $E^{200\text{J}}_f = E^{300\text{J}}_o$?

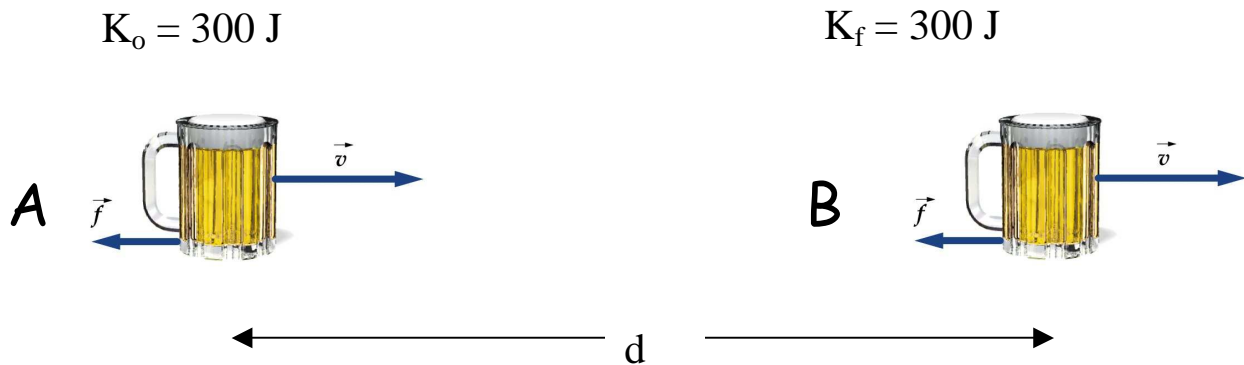
Why not?

$$\Delta E = W_{nc}$$

$$\Delta E = E_f - E_o = 200\text{J} - 300\text{J} = -100\text{J}$$

$\therefore W_{nc} = -100\text{J}$ (Why is the work negative?)

Let's say we observed this for the *same* mug on the *same* rough table:



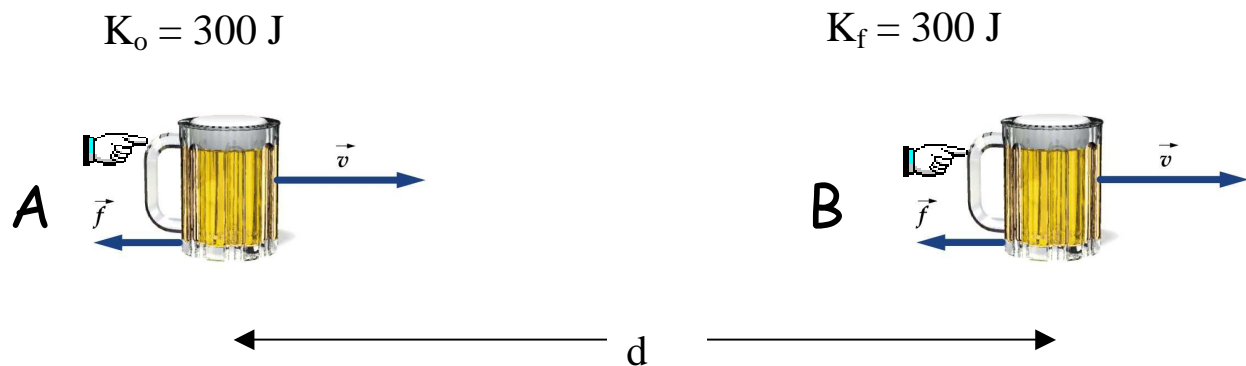
$$\Delta E = W_{nc}$$

$$\Delta E = E_f - E_o = K_f - K_o = 300\text{J} - 300\text{J} = 0\text{J}$$

But $W_{nc} = -100\text{J}$ due to friction, so we have

$$\Delta E = W_{nc} \quad \rightarrow \quad 0\text{J} = -100\text{J} \quad ?!$$

What is wrong here?



There must be another external force doing work! Is this force conservative?

$\Delta E = W_{nc}$ can now be fully accounted for:

$$0\text{J} = -100\text{J}_{\text{friction}} + 100\text{J}_{\text{work done by the hand}}$$

So W_{nc} includes all nonconservative forces.

Thus, we can use this general expression

$$\Delta E = W_{nc}$$

for ALL of our energy problems.