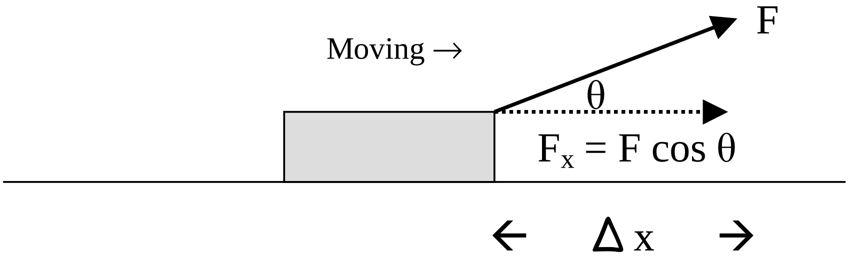
Ch6: Work and Energy

Recall from Ch4: Force: any agent of *change*.

Energy, $\underset{\sim}{E}$, is the capacity a body or system has to cause *change*.

Work is Energy transferred to (+) or removed from (-) a system or body by means of a force acting on it.

Work and Energy are scalars.



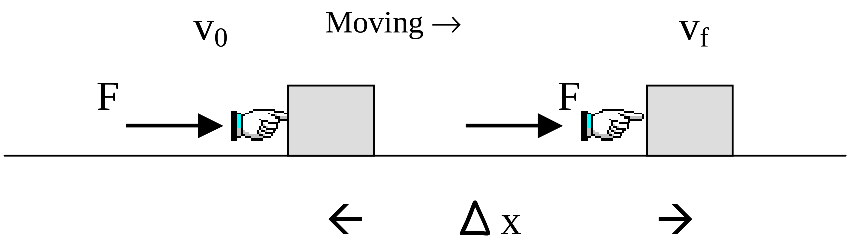
The Work done by the force F on the box is:

Work = (the component of the force along the direction of motion) x (the distance thru which that component acts)

$W = F_x \Delta x$ (units??)

$W = F\cos\theta \; \Delta x \; = \; F \Delta x \; \cos(\vec{F}, \Delta\vec{x})$

For a constant force F:



$W = F_x \Delta x$ but $F = ma$

So $W = ma\,\Delta x$ use the 3[rd] eq of motion…

$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2$

The quantity $\frac{1}{2}mv^2$ is called the
KINETIC ENERGY.

$K = \frac{1}{2}mv^2$

KE is the energy a body or system has due to its motion.

UNITS?? Note: $KE \geq 0$ why?

So, $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2$

becomes $W = K_f - K_o$

or $W_{net} = \Delta K$

Work-Energy Theorem

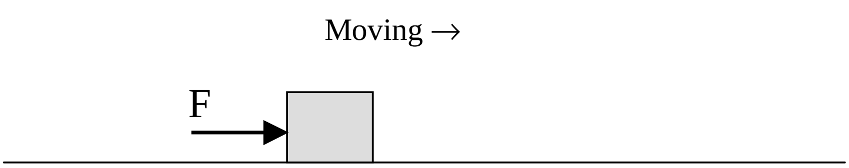
Work-Energy Theorem :

The net work done by all external forces	=	The change in Kinetic Energy

Or, more intuitively:

$$K_f = K_o + W_{net}$$

Thus, when a force F does work on an object, that object will acquire a KE of $\frac{1}{2} m v^2$.



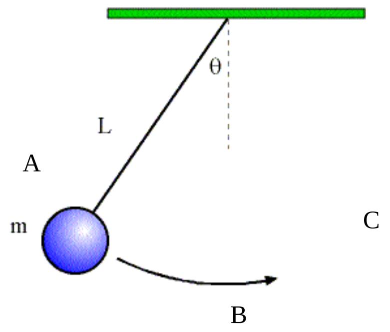
Let's consider the Work done by:

*constant forces
*variable forces

Recap:

Energy: the capacity a body has to cause change.

Work is Energy transferred to or from a body or system by means of a force acting on it.



1. You did work to lift the ball to 'A' by applying a force F.

2. The ball has acquired some form of Energy. (Energy has been transferred to the ball.)

3. Now let go of the ball.

4. The ball speeds up from A -> B because gravity is doing *positive* work.

5. The ball slows down from B -> C because gravity is doing *negative* work.

So we'll see that Work can be both + or -.

Consider a system of objects:
{ Ball + ___?___ }

Let's slowly lift the ball from point A to point B.

$W_{net} = \Delta K$

$W_{net} = W_{gravity} + W_{me} = -mgh + mgh = 0$

So $W_{net} = \Delta K = 0$.

But WORK is the transfer of Energy.

W_{me} = mgh, but in this case the Work done by me did not change the ball's Kinetic Energy, rather the W_{me} is now stored as Gravitational Potential Energy in the Ball-Earth system

GPE: the energy a body has due to its position in a gravitational field.   (Symbol ?)

The ball's Potential Energy at B is called a POTENTIAL energy because that energy is 'potentially' available to be converted/transformed into other forms of energy (…when/how?…)

Potential Energy is an energy due to configuration:

Potential Energy only has meaning w/ respect to some reference point.

Thus only *changes* in Potential Energy have significance.

Let's pick U = 0 at point *A*.

So now ΔU_{AB} = mgh , and we can say:

U_{ball} = mgh at point B.

(*It's really the PE of the ball-Earth system. But since only the ball moves we can say "the PE of the ball". Without the Earth, the ball *has no* PE.)

Now drop the ball:

$W_{gravity,BA} = mgh$, but $\Delta U = -mgh$

So $W_{gravity} = -\Delta U$ (The GPE decreases)

This is true for any Conservative Force.

Gravity is a Conservative Force:

A force is said to be *conservative* if the work done by that force in moving a body from point $A \rightarrow B$ is independent of the path taken from $A \rightarrow B$.

Since $W_{gravity}$ is independent of the path from $A \rightarrow B$, there is some property of the ball-Earth system that depends only on the initial and final positions of the ball.

This property is the GPE.

Also: $\sum W_{conservative} = 0$.

Only conservative forces have potential energy functions:

We had: W = – ΔU (for any conservative force)

But $W = \int \vec{F} \circ d\vec{x}$,or in 1-dimension, $W = \int F(x)\, dx$

So $\Delta U = -\int F(x)\, dx$

This means if we know the conservative force F(x), then we can find its associated potential energy function U(x).

Ex: Find the potential energy function associated with the conservative gravitational force.

$$\vec{F}(y) = mg(-\hat{j}), \quad so\ F(y) = -mg$$

$$\Delta U = -\int_{y_o}^{y} F(y)\,dy = -\int_{y_o=0}^{y} (-mg)\,dy = mgy$$

or $U_f - U_o = mgy$

$U(y) = mgy + U_o$, if we set $U_o = 0$ at y_o , then

$U(y) = mgy$

Potential Energy Stored in a Spring:

$$\Delta U = -\int F(x)\,dx$$, where $F(x) = -kx$

Compression: $$\Delta U = -\int_{0}^{-x} (-kx)\,dx = \frac{1}{2}k\,x^2$$

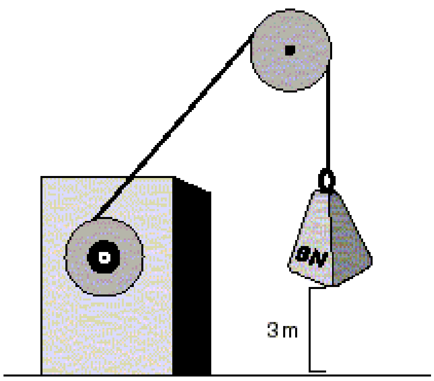
Stretching: $$\Delta U = -\int_{0}^{x} (-kx)\,dx = \frac{1}{2}k\,x^2$$

What if we *had* the PE function, and want to *find* the conservative force associated with the PE function:

$$\Delta U = -\int F(x)\,dx$$

$$dU = -F(x)\,dx\text{, so}\quad F(x) = -\frac{dU}{dx}$$

POWER



Not only do we want to know how much work is needed to lift an object, but we also may want to know *how fast* work is being done (how fast is energy being delivered/transferred/used…).

Power: the rate at which Work is done, or the rate at which energy is transferred.

P = $\frac{Energy}{time} = \frac{dW}{dt}$

Units? $\frac{Energy}{time} \rightarrow \frac{Joules}{\sec ond}$ = Watt, W

So 1 Watt = 1 J/s

$P = \frac{dW}{dt}, \ and \quad W = \int \vec{F} \circ d\vec{x}$, so $dW = \vec{F} \circ d\vec{x}$

then $P = \frac{\vec{F} \circ d\vec{x}}{dt} = \vec{F} \circ \vec{v}$

A motor lifts a 6000-N piano (~ 1300 lbs!) to a height of 8 floors (~ 30 m) in 4 minutes. What is the minimum power that the motor must deliver? (1 hp ~ 746 W)

What do you purchase from SDGE ?