

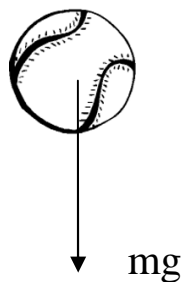
Ch8: Systems of Particles

Center of Mass (CM) \otimes :

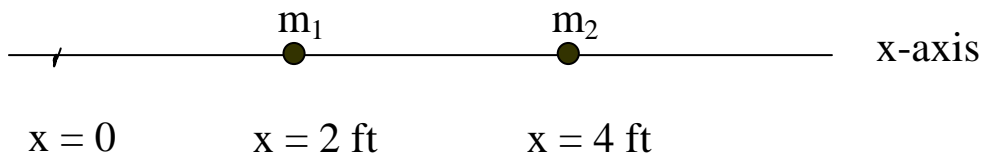
We wish to show that a body or system of particles move as if

1. all of its mass were concentrated at one geometric point in space called the *CM*, and
2. the net external Force was applied there.

How many particles are in this system?



Consider a system of two point-particles.



$$x_{cm} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

This gives us the average position of the particles, weighted by their mass. (It's a weighted average).

If $m_1 = m_2 = m$

Then $x_{cm} = \frac{m(2 \text{ ft}) + m(4 \text{ ft})}{m + m} = 3 \text{ ft}$, just as expected.

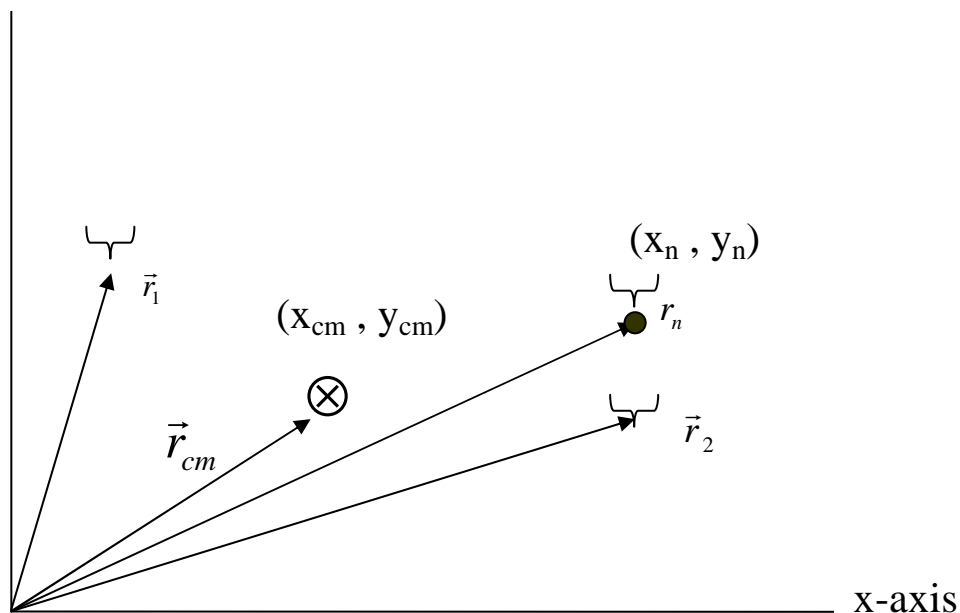
So, for n-particles (discrete masses)

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n}{m_1 + m_2 + m_3 + \dots + m_n} = \frac{\sum_{i=1}^n m_i x_i}{\sum_{i=1}^n m_i}$$

and

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots + m_n y_n}{m_1 + m_2 + m_3 + \dots + m_n} = \frac{\sum_{i=1}^n m_i y_i}{\sum_{i=1}^n m_i}$$

y-axis



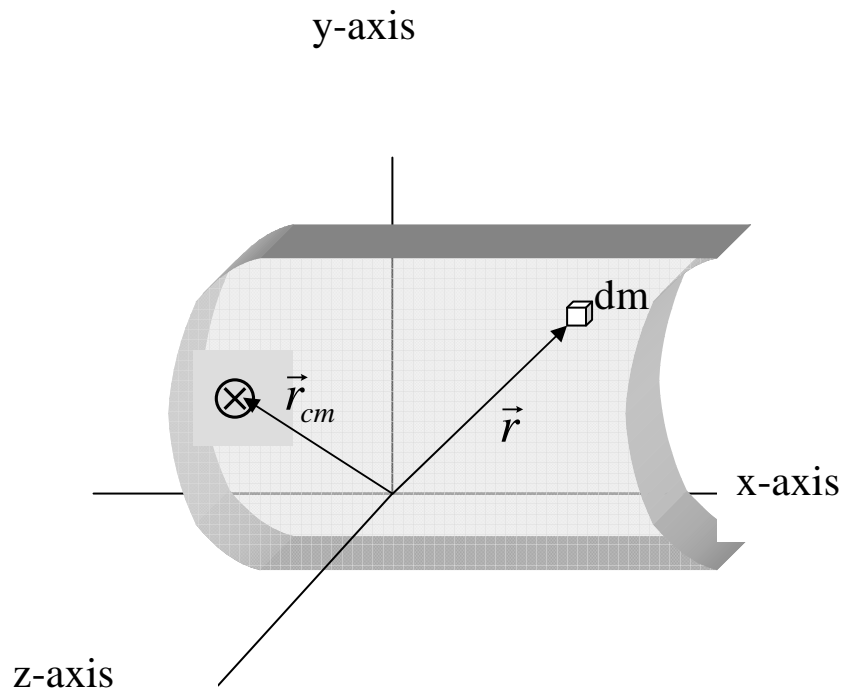
\vec{r}_{cm} = the position of the CM of the system.

$$\vec{r}_{cm} = \frac{\sum_{i=1}^n m_i x_i}{\sum_{i=1}^n m_i} \hat{i} + \frac{\sum_{i=1}^n m_i y_i}{\sum_{i=1}^n m_i} \hat{j}$$

$$\vec{r}_{cm} = \frac{1}{\sum_{i=1}^n m_i} \left(\underbrace{\sum_{i=1}^n m_i (x_i \hat{i} + y_i \hat{j})}_{\vec{r}_i} \right)$$

$$\therefore \vec{r}_{cm} = \frac{\sum_{i=1}^n m_i \vec{r}_i}{\sum_{i=1}^n m_i} \quad \text{for discrete masses (point-particles)}$$

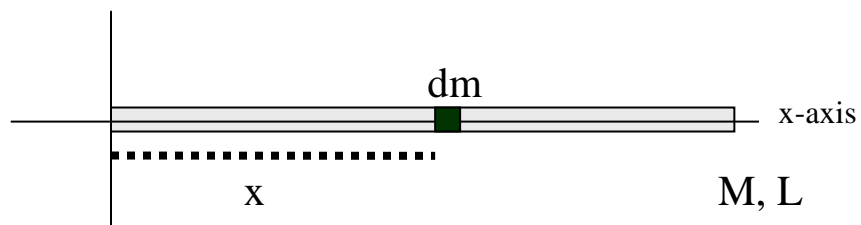
For a continuous distribution:



$$\vec{r}_{cm} = \frac{\int \vec{r} dm}{\int dm}$$

Ex: (1-dimension) Find the position of the center of mass of a thin homogeneous rod of length L and mass M . (ex: a meter stick)

$$x_{cm} = \frac{\int x dm}{\int dm}$$



the linear mass density λ is:

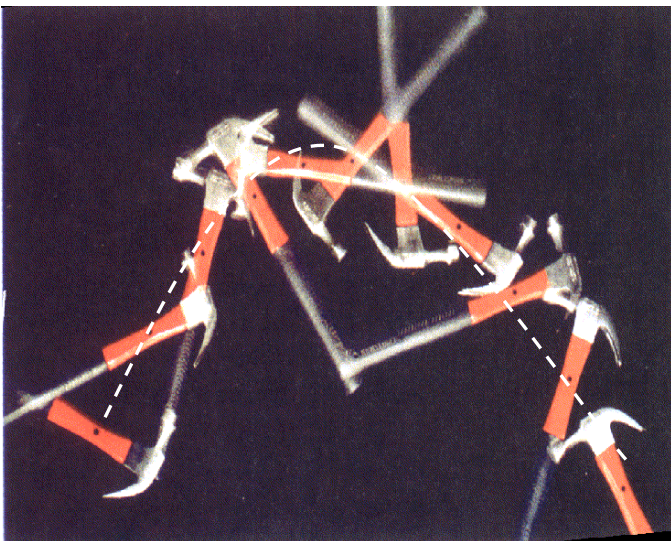
$$\lambda = \frac{\text{mass}}{\text{length}} \quad \lambda = \frac{m}{x} \quad m = \lambda x$$

so $dm = \lambda dx$

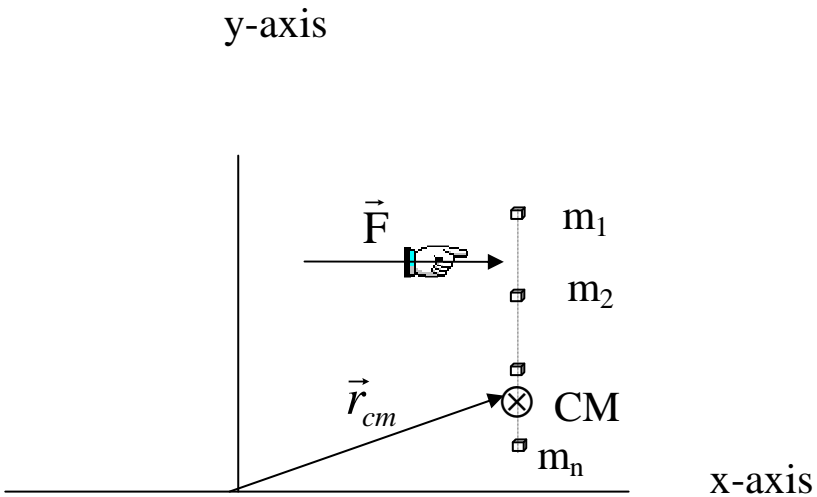
$$x_{cm} = \frac{\int x \lambda dx}{\int \lambda dx} \quad (\text{limits?})$$

We can also find an objects *CM* by hanging it from a string.

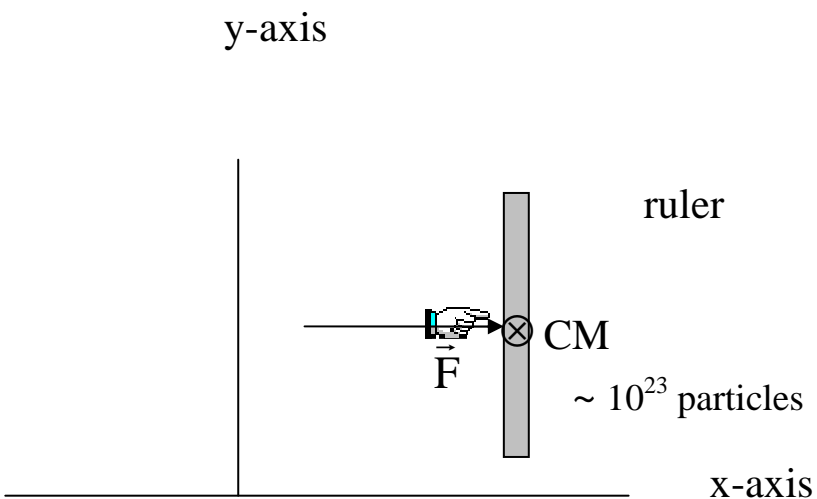
We can show that the *CM* of a body moves according to Newton's 2nd Law:



To do this, consider a system of n -particles
 (connected by tiny, massless, rigid rods) on which a quick
 force \vec{F} acts.



How would this system of particles move?
 What if n was large?



Is $\Sigma F = ma$? Which m ? Which a ?