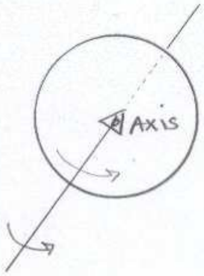
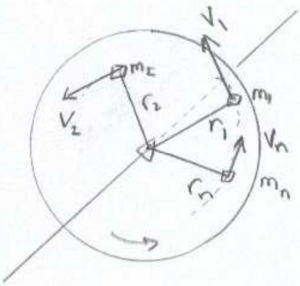
Rotational Kinetic Energy



Rotating wheel.

Is $K = \frac{1}{2} m v^2$?

Let's break up the wheel into tiny chunks of mass:



$$K_{rot} = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 + \dots + \frac{1}{2} m_n V_n^2$$

$$K_{rot} = \sum \frac{1}{2} m_i V_i^2 \quad , \quad V_i = r_i \omega$$

so $K_{rot} = \frac{1}{2} \sum m_i (r_i \omega)^2 \implies K_{rot} = \frac{1}{2} \underbrace{\left(\sum m_i r_i^2 \right)} \omega^2$

$\sum m_i r_i^2$ is a measure of both the mass & how how the mass is distributed about the axis of rotation.

$I = \sum m_i r_i^2 \equiv$ the Moment of Inertia or "Rotational Inertia" of the object about the axis of rotation.

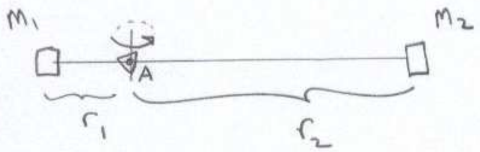
$\therefore K_{rot} = \frac{1}{2} I \omega^2$

Compare to $K_{\text{Translational}} = \frac{1}{2} m v^2$

$\underline{I} = \sum m_i r_i^2$, is a measure of a body's resistance to $\underline{\text{rotate}}$ about some specified axis.

We'll treat I as a $\underline{\text{scalar}}$ in this class. $\{$ units? $\}$

Ex) 2 point-masses.



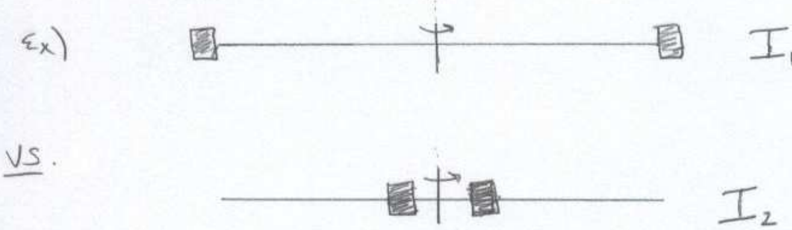
$$I_A = m_1 r_1^2 + m_2 r_2^2$$

How is "$\underline{I}$" related to "Inertia" from Chapter 4?

Inertia = a measure of a body's resistance to move due to its mass.

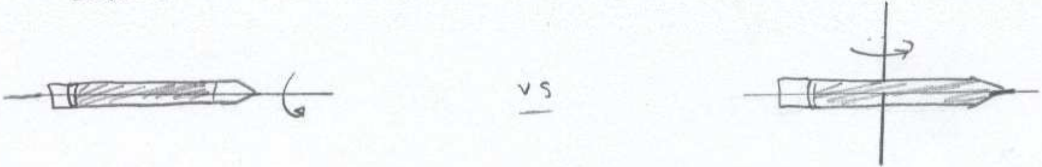
Moment of Inertia, I = a measure of a body's resistance to ROTATE about some specified axis due to its mass distribution about that axis.

Ex)



Compare I_1 & I_2. Which is 'easier' to twist?

Try rotating your pencil about 2 different axes:



Which case offers more resistance (I_{Big}) to Rotate?